Exponents

An integer is multiplied by itself one or more times.

The integer is the base number and the exponent (or power). The exponent tells how many times the base number is multiplied by itself.

Example: 4^3 4 is the base number, 3 is the exponent.

$$4^3 = 4 \times 4 \times 4$$
$$= 64$$

When an integer is to the power of 2, it can be expressed as squared.
When an integer is to the power of 3, it can be expressed as cubed.

Rule:
Any number to the first power is that number.
Example: $8^1 = 8$.

Rule:
Any number to the power of 0 is 1.
Example: $6^0 = 1$

Products of Powers with Like Bases

A product of powers with like bases (same base number) can be simplified by adding exponents.

Example: $3^4 \times 3^2$ $\quad 3^4 = 3 \times 3 \times 3 \times 3$ $\quad 3^2 = 3 \times 3$
$3^{4+2} = 3^6$ $\quad 3^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$

Rule:
To multiply numbers with the same base, add the exponents.

Quotients of Powers with Like Bases

Division also may involve like bases. A quotient of powers with like bases can be simplified by subtracting the exponents.

Example: $4^7 \div 4^5$
$= 4^{7-5}$
$= 4^2$ $4^2 = 4 \times 4$

Rule:
To divide numbers with the same base, subtract the exponents.

Suggested Practice: Number Power 3 pages 27-29

BEDMAS

It is an acronym to help remember the order of operations in math problems that require the use of different operations.

Brackets
Exponents
Division
Multiplication
Addition
Subtraction

The operations are completed in that order. Brackets first, Exponents second, Division or Multiplication third (in the order they appear), and lastly Addition or Subtraction (in the order they appear).

Example: $(2 \times 7) - 2^2 \div 2$

1. Brackets $2 \times 7 = 14$
   $14 - 2^2 \div 2$
2. Exponents $2^2 = 2 \times 2 = 4$
   $14 - 4 \div 2$
3. Division $4 \div 2 = 2$
   $14 - 2$
4. Subtraction $14 - 2 = 12$

Answer is 12
Suggested Practice: Number Power 3 pages: 72-75, 78-79

Fractions

ADDING AND SUBTRACTING

To add and subtract fractions, the **denominator (bottom number) must be** the same.

1/5
+ 3/5
= 4/5 Add the top only, bottom number stays the same.

RAISING AND LOWERING **"What you do to the bottom, you must do to the top."**

To raise a fraction up, ***multiply*** the top and bottom number by the same number.

To reduce a fraction, ***divide*** the top and bottom number by the same number.

To Change a Mix Number to an Improper Fraction

Take the denominator multiply by the whole number and add the numerator.

Example: 32/5 – 4/5

$$3\,2/5 = \frac{5 \times 3 + 2}{5} - 4/5$$

17/5 – 4/5 = 13/5

Now reduce: How many times does 5 go into 13?
23/5

MULTIPLYING Denominators (bottom number) do ***Not*** have to be the same.

Multiply straight across the top and straight across the bottom. Remember to turn mixed fractions into improper ones before multiplying.

Example: 3 2/5 turns to 17/5

Reduce your answer first by cancelling, then multiply OR it will still work without cancelling. (to review cancelling, ask your instructor!)

DIVIDING Denominators (bottom number) do ***Not*** have to be the same.

Invert the second fraction (eg. 2/3 turns to 3/2) and multiply. (see multiplying rules above!)

Example: 4/5 ÷ 2/3

4/5 X 3/2 = 12/10 = 1 2/10 = 1 1/5 (reduced to lowest terms)

Remember to turn mixed fractions into improper ones before dividing. (eg. 32/5 turns to 17/5)

Always, Always, Always **REDUCE YOUR ANSWER TO ITS LOWEST TERMS**!!

To represent a whole number as a fraction it is written with the denominator of 1. Example: 3 written as a fraction is 3/1.

Suggested Practice: Number Power 2 Pages: For Adding 26-28, For Subtracting 32-33, For Multiplying 39, 41, 43 and for Dividing 48, 52, 53.

Percentage

Percent is one more way - besides fractions and decimals - of describing a part of a whole.

Percentages are always "out of" one hundred. 35% means 35/100, 66% means 66/100.

Changing Decimals to Percents

Move the decimal point 2 places to the right (bigger) and insert the percent sign.

Example: .44 = 44% .07 = 7% .0006 = .06%

Changing Percents to Decimals

Move the decimal point in the percent 2 spaces to the left (smaller) and remove percent sign.

Eg. 6% = .06 30% = .3 150% = 1.5

Changing Fractions to Percents

Divide the bottom number into the top number and move the decimal point 2 places to the right.

Example: ¾ = 3 ÷ 4 = .75 Change this decimal to a percent = 75%.

Changing Percents to Fractions

Write the percent as a fraction (% sign means over 100) and REDUCE.

Example: 85% = 85/100 = 17/20

Find the Percent of a Number

Change the percent to a decimal (move decimal point 2 places to the right) then multiply with the number.

Example: Find 25% of 80. 25% = .25 X 80 = 20.

OR using a calculator, enter 80 X 25 and push the percent (%) button = 20.

Find What Percent One Number is of Another Number

9 is what percent of 45?

Example: Put 9/45, reduce to 1/5. Divide bottom number into top number = .20

Change decimal .20 to 20% by moving decimal point 2 places to the right and add the % sign.

Suggested Practice: Number Power 2 Pages: 98-102, 104, 107

Mean, Median and Mode

Mean is the average of a series of numbers. To find the average add all the numbers and divide by the number of values.
Example: 13, 18, 13, 14, 13, 16
13 + 18 + 13 +14 + 16 = 87
87 ÷ 6 = 14.5
The average is 14.5.

Median is the middle value. To find the median the numbers have to be listed in numerical order. Cross out the numbers until you reach the middle.
Example: 13, 18, 13, 14, 13, 16
Since there is no middle number, take the two middle numbers and find the average.
13 + 14 = 27
27 ÷ 2 = 13.5
The middle number is 13.5

Mode is the number that occurs most often.
Example:
13, 13, 13, 14, 16, 18
Mode = 13

Range is the difference between the largest and smallest values.
Example: 13, 18, 13, 14, 13, 16
18 – 13 = 5
The range is 5.
Suggested Practice: Online at Khan Academy. Search mean, median and mode. Optional videos and practice questions to complete.

Polynomials

-Means many terms. Polynomials have constants (like 3, -20, or ½), variables (like x and y) and exponents (like y^2) that can be combined using addition, subtraction, multiplication and division.

Exception: No division by a variable. Example: 2/x.

Example: $4xy^2 + 3x - 5$
This polynomial has 3 terms.

Rules for Adding and Subtracting Signed Numbers:

- When working with positive and negative numbers, the answer or product will have the same sign as the largest number.
- When subtracting, remember two negatives equals a positive. Example: $6z - (-3z) = 6z + 3z = 9z$

Rules for Multiplying and Dividing Signed Numbers

- If the signs of the numbers are alike, multiply or divide the numbers and give the product(answer) a positive sign. Examples: negative x negative = positive, positive ÷ positive = positive
- If the signs of the numbers are different, multiply or divide the numbers and give the product a negative sign. Examples: negative ÷ positive = negative, negative x positive = negative

Examples:
A. $3x + 2x + 4y$
$(3 + 2 = 5)$
$= 5x + 4y$
x and y are unlike terms

B. $-5a^2b + a^2b$
$(-5 + 1 = -4)$
$= -4a^2b$
The coefficient of a^2b is 1.

C. $(6x - 9y) + (-5x + 4y)$
Since there are no like terms in the parentheses, remove the parentheses.

$6x - 9y + (-5x) + 4y$

Group like terms to simplify.

$6x + (-5x) - 9y + 4y$

$(6 + -5 = 1)$ $(-9 + 4 = -5)$

$= x - 5y$

Suggested Practice: Number Power 3 pages 130-133

Distributive Property

Is an algebra property which is used to multiply a single term and two or more terms inside the parentheses.

Examples:

$$5\,(x + 2) = 5 \bullet x + 5 \bullet 2$$

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$$(5x)(3x + 6) = 5x \bullet 3x + 5x \bullet 6$$

$$(5)(3x^2 + 2x + 6) = 5 \bullet 3x^2 + 5 \bullet 2x + 5 \bullet 6$$

• means to multiply, and when two terms are in parentheses it also indicates multiplication.

Answers:

$5 \bullet x + 5 \bullet 2 = 5x + 10$ »no like terms, cannot simplify any further.

$5x \bullet 3x + 5x \bullet 6 = 15x^2 + 30x$

$5 \bullet 3x^2 + 5 \bullet 2x + 5 \bullet 6 = 15x^2 + 10x + 30$

Suggested Practice: Number Power 3 pages: 131, 133-136, 139.

Graphing

X is the horizontal axis.

Y is the vertical axis.

Positive x coordinates indicates the point is to the right of the y-axis.
Negative x coordinates indicates the point is to the left of the y-axis.
Positive y coordinates indicates the point is above the x axis.

Negative y coordinates indicates the point is below the x axis.
Coordinates are usually written as an ordered pair, two numbers in parentheses. The x-coordinate (x-value) is written first followed by the y-coordinate (y-value).
Example: (-6, 4)
-6 is the x-coordinate, 4 is the y-coordinate.

Suggested Practice: Number Power 3 pages: 100-103, 106-107, 112-115.

Slope

Going up or down a hill is walking on a slope.

The slope of a line is given as a number. When you move between 2 points on a line the slope is found by dividing the change in y-value by the corresponding change in the x-value.

$$\text{Slope of a line} = \frac{\text{change in y-value (rise)}}{\text{Change in x-value (run)}}$$

Suggested Practice: Number Power 3 pages: 104-105.

Solving Equations Mathematically

A few hints to solve equations mathematically:
- Remember the importance of keeping the equation "balanced" like with Hands-On-Algebra.
- Think of "undoing" like with the flow charts.

"**UNDO**" *adding* by *subtracting.* "**UNDO"** *subtracting* by *adding*.
"**UNDO**" *multiplying* or *dividing. "***UNDO"** *dividing* by *multiply.*
Examples:

5 +3g =23
What to "undo" first?
Undo adding 5 by subtracting 5. Keep the equation balanced by subtracting 5 from both sides.

$$\begin{array}{rcl} 5 + 3g &=& 23 \\ \underline{-5 \quad\quad} && \underline{-5} \\ 3g &=& 18 \end{array}$$

Next, since we are solving for g, g needs to be isolated. So undo 3 x g by dividing both sides by 3.

$$\frac{3g}{3} = \frac{18}{3}$$

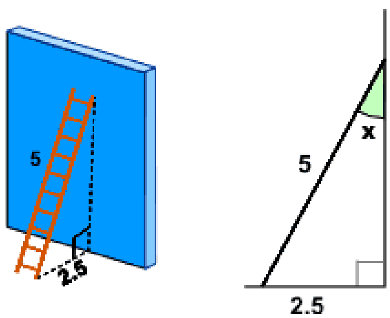
g = 6

6 - 5p = p + 30
-6   - 6   Take 6 from each side.
-5p = p + 24
-p = - p   Take 1p from each side.
-6p = 24
-6p = 24   Divide both sides by -6.
-6 = -6
p = -4

Suggested Practice: Number Power 3 pages: 69, 73, 75, 79.

Trigonometry

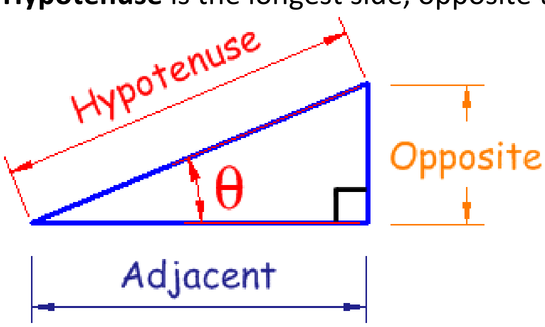
Finding an unknown angle in a right-angled triangle, you use the known length of two of its sides.
Example:
A 5 foot ladder leans against a wall. What is the angle between the ladder and the wall?



The answer is to use Sine, Cosine or Tangent.

Which one to use?
Step I: Find the names of the two sides you know.

Adjacent is adjacent (beside) the angle.
Opposite is opposite the angle.
Hypotenuse is the longest side, opposite the right angle.



In the example, we know the length of the side opposite angle x (2.5ft) and the longest side opposite the right angle, called the hypotenuse (5ft).

Step II: To find out which one of Sine, Cosine or Tangent to use:

SOH Sine: $\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$

CAH Cosine: $\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

TOA Tangent: $\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}$ A way to remember is "SOHCAHTOA."

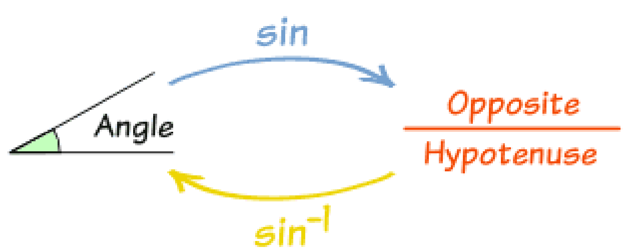
Step III: Put the values into the Sine equation, since we know the length of the opposite side to angle x and the length of the hypotenuse.

Sin(x) = Opposite/Hypotenuse = 2.5/ 5 = 0.5

Step IV: Solve the equation.
Sin (x) = 0.5
This can be rearranged
$x = \sin^{-1}(0.5)$
With a scientific calculator, key in 0.5 and use the $\sin^{-1}$ button to get the answer. Calculator has to be in degree mode.
$x = 30^0$

Note: To use $\sin^{-1}$ you would press either '2ndF sin' or 'shift sin.' It is the same to use $\cos^{-1}$ or $\tan^{-1}$.

What is $\sin^{-1}$?
Well, the Sine function "sin" takes an angle and gives us the ratio "opposite/hypotenuse."



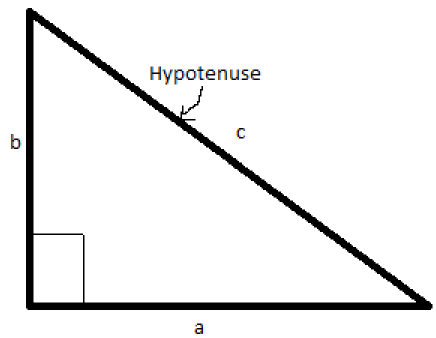
But $\sin^{-1}$ (called "inverse sine") goes the other way….it takes the ratio "Opposite/Hypotenuse" and gives us an angle.

Example: Sine Function: $\sin(30^o) = 0.5$
Inverse Sine Function: $\sin^{-1}(0.5) = 30^o$

Answer: $x= 30^o$
The angle between the ladder and the wall is 30°.
Suggested Practice: Online at Khan Academy search trigonometry. Option to watch videos and complete practice questions.

Pythagorean Theorem

The Pythagorean Theorem applies to a triangle where two sides meet at a right angle (90^0). The side opposite the right angle is called the hypotenuse.



The Pythagorean Theorem was discovered by a Greek mathematician Pythagoras who found that the square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the other two sides. Using the labels in the triangle above.

Pythagorean Theorem: $c^2 = a^2 + b^2$ in words hypotenuse squared equals side squared plus side squared.

Example of Pythagorean Theorem:

Find the length of the hypotenuse in the triangle below.

Pythagorean Theorem: $c^2 = a^2 + b^2$

Step I: Substitute 3 for ***a*** and 4 for ***b*** in the Pythagorean Theorem.

$c^2 = a^2 + b^2$ $\quad\quad$ $3^2 = 3x3$

$c^2 = 3^2 + 4^2$ $\quad\quad$ $4^2 = 4x4$

= 9 + 16

= 25

Step II: Solve for **c**.

c2 = 25

$c = \sqrt{25}$

c = 5mm

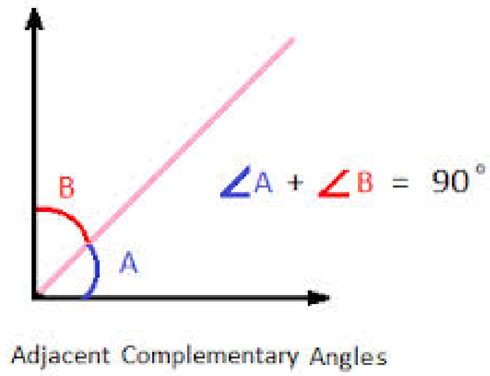
Answer: The length of the hypotenuse is 5mm.

Suggested Practice: Number Power 4 pages: 65, 67, 69.

Angles

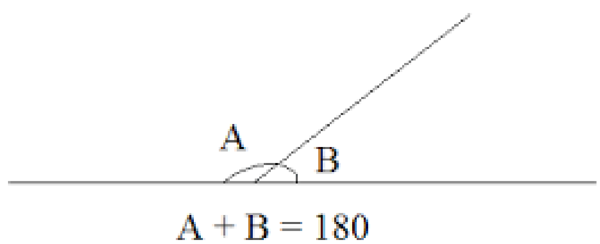
Complimentary Angles are angles whose sum is 90^0.

To find the value of a complementary angle, subtract the known angle from 90^0.

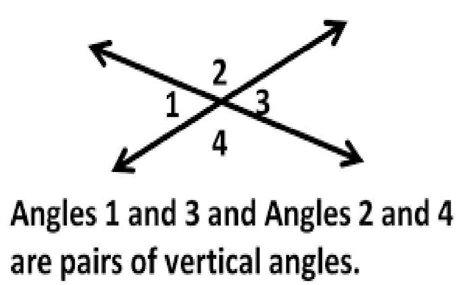


Supplementary Angles are angles whose sum is 180^0.

To find the value of a supplementary angle, subtract the known angle from 180^0.



Vertical Angles are formed when two straight lines cross at a point. Vertical angles lie across from each other and are equal.



Suggested Practice: Number Power 4 pages: 27, 29, 31, 36.

Triangles:

All 3 angles in a triangle equal 180^0.

<u>Equilateral Triangle</u>
-has three equal angles, and all three sides are equal.

<u>Isosceles Triangle</u>
-has two equal angles. The sides opposite the equal angles are also equal. The two equal angles are called **base angles**. The third angle is called the **vertex angle.**

<u>Scalene Triangle</u>
-has no equal sides and no equal angles. Subtract the two known angles from 180^0.

Suggested Practice: Number Power 4 pages 50-51.

Factoring

An algebraic expression can often be written as a product of factors. If each term of an expression can be divided evenly by a number or variable, that number or variable is a factor of the expression.

Factoring Out a Number

- Find the greatest number that divides evenly into each of them
- Write the number (that divides evenly) on the outside of a set of parentheses.

Example: Factor 4x + 10
What divides evenly? 2
=2 (2x + 5)

Factoring Out a Variable
An algebraic expression where a number does <u>not</u> evenly divide into each term, but a variable <u>does</u> evenly divide into each term.

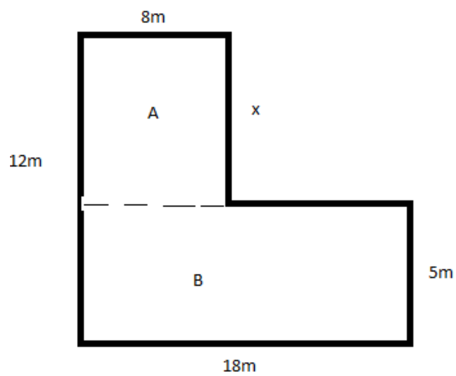
Example: $5x^2 + 3x$
- Each term can be divided evenly by x. So, x can be factored out.
- Write x on the outside of a set of parentheses. Divide each term by x.

$5x^2 \div x = 5x$
$3x \div x = 3$
$=x(5x + 3)$

Suggested Practice: Number Power 3 pages: 150-153, 146-148.

Area

Finding the area of a two shaped figure, like an L shaped room.



Step I: Divide the shape into two rectangles.
Step II: Find the unknown side of the one rectangle.
Step III: Find the area of each rectangle.
Step IV: Add the areas of the two rectangles together.

Step I is represented by the dotted line.

Step II: To find x subtract 5m from 12m
X= 12m-5m
X= 7m

Step III: To find the area of each rectangle label one rectangle A and the other rectangle B.
Area A: length x width= l x w
= 7m x 8m
= $56m^2$
Area B: l x w
= 18m x 5m
= $90m^2$

Step IV: Add the area of rectangle A and B
$56m^2$
$\underline{+ 90m^2}$
$146m^2$
Answer: The area of the room is $146m^2$.

Suggested Practice: Number Power4 pages: 115-116, 119-121.